Probabilistic Method

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June 3, 2015

Introduction

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- It is a nonconstructive method as you show something exist without constructing an actual object.
- Szele in 1942 was the first one to apply this method successfully.
- However it was Paul Erdős who popularize this method.

Hamiltonian Paths

Theorem (Szele,1943)

There is a tournament T with n players and at least $\frac{n!}{2^{n-1}}$ Hamiltonian paths.

Ramsey Number, $R(k, k)$

Definition

For fix positive integers k, l, the number N is called the **Ramsey number,** $R(k, l)$ if in the two edge colorings (say red and blue) of complete graph K_N on N vertices, there must exist a complete subgraph on k vertices with all red edges or a complete subgraph on l vertices with all blue edges.

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Example:
$R(3, 3) = 6, R(4, 4) = 18, R(5, 5) \in [43, 49], R(6, 6) \in [102, 165]$.

Ramsey Number, $R(k, k)$

Figure: The prince of problem posers, Paul Erdős in 1992

Famous Quote: Suppose aliens invade the earth and threaten to obliterate it in a year's time unless human beings can find the Ramsey number for red five and blue five. We could marshal the world's best minds and fastest computers, and within a year we could probably calculate the value. If the aliens demanded the Ramsey number for red six and blue six, however, we would have no choice but to launch a preemptive attack.

Ramsey Number, $R(k, k)$

Theorem (Ramsey,1929)

$R(k, l)$ is finite for any two integers k and l.
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Theorem (Erdős,1947)

$R(k, k) > \left\lfloor 2^{\frac{k}{2}} \right\rfloor$ for all $k \geq 3$.

Ramsey Number, $R(k, k)$

Olympiad Corner: Show that among 2^{100} people, they do not necessarily exist 200 people who know each other or 200 people who don't know each other.