MATH 373: CLASS 19

1. EXERCISE

1) Show each of the following initial-value problems has a unique solution.

    a) $y' = y * \cos(t), \quad 0 \le t \le 1, \quad y(0) = 1$.

    b) $y' = \frac{2}{t}y + t^2 e^t, \quad 1 \le t \le 2, \quad y(1) = 0$.

2) Given $y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1$ with $h = \frac{1}{2}$.

    a) Use Euler's Method to approximate $y(\frac{5}{2})$ and $y(3)$.

    b) Find Lipschitz constant.

    c) In addition, suppose we know the solution $y(t) = t + \frac{1}{1-t}$. Find $max_{t \in [2,3]} |y''(t)|$.

    d) Find an error bound at $t = \frac{5}{2}$ and $t = 3$.

    e) Compare an actual error with an error bound.