MATH 373: CLASS 17

1. Exercise

1) Let $S(a,b)$ be the Simpson's rule approximation of $\int_a^b f(x)dx$.

Estimate $\left|\int_0^{\frac{\pi}{4}} x^2 sin(x)dx - S(0,\frac{\pi}{8}) - S(\frac{\pi}{8},\frac{\pi}{4})\right|$.

2) Let $T(a,b)$ be the trapezoidal rule approximation of $\int_a^b f(x)dx$.

Derive the relation between $\left|T(a,b) - T(a,\frac{a+b}{2}) - T(\frac{a+b}{2},b)\right|$
and $\left|\int_a^b f(x)dx - T(a,\frac{a+b}{2}) - T(\frac{a+b}{2},b)\right|$.

3) Find c_1, c_2, x_1 and x_2 so that $\int_{-1}^{1} f(x)dx \simeq c_1 f(x_1) + c_2 f(x_2)$ has degree of precision $2*2-1=3$.

4) Let $P_n(x)$ be Legendre polynomial of degree n which defined by

$P_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2-1)^n}{dx^n}$.

Show $P_n(1) = 1$ and $P_n(-1) = (-1)^n$ for all positive integer n.

(Hint : use definition.)

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