NUMBER THEORY: CLASS 7

1. Exercise

1) Find the prime factorizations of each of the following integers:
a) $2^{15} - 1$
b) $2^{24} - 1$
c) $2^5 + 1$

2) Find the generating function for the sequence a_n where a_n is the number of ways to make change for n cents using only nickels, dimes and quarters.

3) Find the generating function for the sequence a_n where a_n is the number of integers solutions to the equation

$$x_1 + 2x_2 + 3x_3 + 4x_4 = n$$

with x_1, x_2, x_3, x_4 are nonnegative integer.

4) a) Find the generating function for the sequence a_n where a_n is the number of ways to make change for n cents using only coins.

b) Use maple to solve for the number to ways to make change for 5 dollars.

In Maple program:

First, define your generating function. For example

$P := 1/(1-x)(1-x^5)(1-x^{10})(1-x^{25});$

Second, use Taylor expansion around $c = 0$, try:

$T := taylor(P, x = 0, 501);$

Finally try to find the coefficient to x^{501}. You can look up the help command in Maple by typing:

$?coeff;$

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