NUMBER THEORY: CLASS 5

1. Exercise

1) In this problem, we will rediscover the statement of the Prime Number Theorem using Maple Program:

I) Command Lines

Some useful commands about primes.

i) isprime(n):

input: number n
output: true if n is a prime and false otherwise

try: $isprime(2532490438063)$;

ii) ithprime(n)

input: number n
output: n^{th} prime

try: $ithprime(4)$;

iii) numtheory[pi](n)

input: number n
output: the number of prime less or equal to n

try: $numtheory[pi](100)$;

II) Other functions:

i) To find value of $\frac{x}{log(x)}$ try

$evalf(10/ln(10))$;

ii) To find value of $Li(x) := \int_2^x \frac{1}{ln(t)} dt$ try

$evalf(int(1/ln(t), t = 2..10^{14}))$;

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Problem

a) Fill up the numbers in the table below:

n	$\pi(n)$	$\frac{n}{ln(n)}$	$\frac{n}{\pi(n)ln(n)}$	$Li(n)$	$\frac{Li(n)}{\pi(n)}$
10					
10^3					
10^9					
10^{15}					
10^{21}					

FIGURE 1

b) Which function , $\frac{n}{log(n)}$ or $Li(n)$, is a better estimation of $\pi(n)$?

2) Show there are infinitely many prime of the form $6n + 5$.

3) Show that $\sqrt{2} + \sqrt{3}$ is irrational.