NUMBER THEORY: CLASS 15

1. Exercise

1) What is the remainder when 16! is divided by 19?

2) I) List all the positive numbers less than n that are relatively prime to n.
II) Check the numbers of elements in each part with $\phi(n)$.

a) $n = 12$, b) $n = 16$, c) $n = 25$.

3) Solve for the remainder when 7^{98} is divided by 15.

4) Given $gcd(b, m) = 1$ and a and c are positive integers.

Prove: if $p \mid (b^n + 1)$ then either

i) $p \mid (b^d + 1)$ for some proper divisor d of n for which n/d is odd, or
ii) $p \equiv 1 \bmod 2n$.

5) Use the result in problem 4 to conclude that

if $p \mid 2^{2^n} + 1$ then $p \equiv 1 \bmod 2^{n+1}$.

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