NUMBER THEORY: HOMEWORK 7

Homework due on Friday November 7.

1. Problems

1) Use the program posted on the website to answer the followings:

a) Find all positive integers $n \leq 50000$ that is a strong pseudoprime to the base 2.
b) Fin all positive integers $n \leq 50000$ that is a strong pseudoprime to the base 3.
c) How many $n \leq 50000$ such that n is a strong pseudoprime to both bases 2 and 3.

2) Prove that there are infinitely many integers n for which $\phi(n)$ is a perfect square.

3) Prove that $\tau(n)$ is odd if and only if n is a perfect square.

4) Prove that if n is a perfect number, then $\sum_{d|n} \frac{1}{d} = 2$.

5) Prove that $\tau(n) < 2\sqrt{n}$.

6) Prove that $\sigma(n) \equiv \tau(m) \pmod 2$ where m is the largest odd factor of n. Hint: First consider $n = p^\alpha$ for $\alpha \geq 0$.

Also do the following problems:

Problem 8 page 245.
problem 26, 27 page 267.

Date: Friday, October 31, 2008.