May 1, 2009

Calculus II : Review Problems for Final Exam

Extra 3% of total grade

Due Saturday, May 9th 2009 before midnight (My office: 241 Tome)

Section 5.8: l'Hospital's Rule
Evaluate the following limits

1. $\lim_{x\to 0^+} \dfrac{\ln x}{x^2}$.
2. $\lim_{x\to 1} \dfrac{x^9-1}{x^5-1}$.
3. $\lim_{x\to 0} \dfrac{\arcsin x}{x}$.
4. $\lim_{x\to 3} \dfrac{x+2}{x^2-16}$.
5. $\lim_{x\to 0} \dfrac{\tan x - x}{x^3}$.

Section 6.1, 6.2 and 6.3: Integration Techniques
Evaluate the following integrals

1. $\displaystyle\int (\ln x)^2\, dx$.
2. $\displaystyle\int \arctan(\frac{1}{x})\, dx$.
3. $\displaystyle\int (1+\cos x)^2\, dx$.
4. $\displaystyle\int \frac{\sqrt{x^2-1}}{x}\, dx$.
5. $\displaystyle\int x\sqrt{x^2+4}\, dx$.
6. $\displaystyle\int \frac{x^2}{x^2-4}\, dx$.
7. $\displaystyle\int \frac{x}{x+5}\, dx$.
8. $\displaystyle\int \frac{1}{x^2(x-1)^2}\, dx$.

Section 7.1, 7.2 and 7.3: Area between curves and volume

1. Find the area enclosed by the line $y = x + 4$ and the parabola $y = x^2 - 2x$.
2. Find the area enclosed by the curves $y = x^3 - x$ and $y = 3x$.
3. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $y = x$, $x \geq 0$ about the x-axis.
4. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$, $x = y^2$ about the line x = -1.
5. Find the volume of the solid obtained by rotating the region bounded by the curves $x = 1 + (y - 2)^2$ and $x = 2$ about the x-axis.

Section 8.2, 8.3 and 8.4: Series

Determine whether the series is absolutely convergent, conditionally convergent, or divergent

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + 3}$.
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3 + 3}$.
3. $\displaystyle\sum_{n=1}^{\infty} \frac{n + 4^n}{2n + 3^n}$.
4. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n}$.
5. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$.
6. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^3}$.
7. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$.
8. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$.

Section 8.5, 8.6 and 8.7: Power Series

1. Find the radius of convergence and interval of convergence of the series
$$\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}.$$
2. Find the radius of convergence and interval of convergence of the series
$$\sum_{n=1}^{\infty} n!(2x-1)^n.$$
3. Find the Maclauring series of $f(x) = \dfrac{x^2}{x^2+16}$.
4. Find the Maclaurin series of $f(x) = xe^{-x^2}$.
5. Find the power series representation of $f(x) = (\cos x)^2$ (Hint: $(\cos x)^2 = \frac{1}{2}(1+\cos(2x))$.
6. Find the power series representation of $f(x) = x^3 + 2x$ centered at $a = -1$.