February 9, 2009

# Lab 4: Chapter 5 Review

TEAM MEMBERS

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*INSTRUCTIONS:* Work the following problems and write up your solutions neatly, clearly, and carefully. All members of the team should understand and be able to explain the solutions.

**I**. The following are True/False questions. Write "True" or "False" and explain why.

1. If $f$ is one-to-one, with domain $\mathbb{R}$, then $f^{-1}(f(6)) = 6$.

2. The function $f(x) = \cos x$, $-\pi/2 \leq x \leq \pi/2$, is one-to-one.

3. $\tan^{-1}(-1) = 3\pi/4$.

4. If $0 < a < b$, then $\ln a < \ln b$.

5. $\pi^{\sqrt{5}} = e^{\sqrt{5}\ln\pi}$.

6. You can always divide by $e^x$.

7. If $a > 0$ and $b > 0$, then $\ln(a + b) = \ln a + \ln b$.

8. If $x > 0$, then $(\ln x)^6 = 6 \ln x$.

9. $\dfrac{d}{dx}(10^x) = x10^{x-1}$.

10. $\dfrac{d}{dx}(\ln 10) = \dfrac{1}{10}$.

11. The inverse function of $y = e^{3x}$ is $y = \frac{1}{3} \ln x$.

12. $\cos^{-1} x = \dfrac{1}{\cos x}$.

13. $\lim\limits_{x \to \pi^-} \dfrac{\tan x}{1 - \cos x} = \lim\limits_{x \to \pi^-} \dfrac{\sec^2 x}{\sin x} = \infty$.

**II**. The following require a short answer.

1. (a) What is a one-to-one function?

   (b) How can you tell if a function is one-to-one by looking at its graph?

   (c) How do you obtain the graph of $f^{-1}$ from the graph of $f$?

2. (a) What are the domain and range of the natural exponential function $f(x) = e^x$?

   (b) What are the domain and range of the natural logarithmic function $f(x) = \ln x$?

   (c) How are the graphs of these functions related?

   (d) If $a$ is a positive number, $a \neq 1$, write an equation that expresses $\log_a x$ in terms of $\ln x$.

3. (a) What are the domain and range of $\arcsin x$?

(b) What are the domain and range of $\arccos x$?

(c) What are the domain and range of $\arctan x$?

(d) Sketch the graph of $\arctan x$.

4. Compute the derivatives of the following functions.

(a) $y = e^x$.

(b) $y = a^x$.

(c) $y = \ln x$.

(d) $y = \ln(\sin x)$

(e) $y = \arctan(x^2 + 6x)$

(f) $y = x \ln |x - e^{x^2}|$

5. l'Hospital's Rule

(a) What does l'Hospital's Rule say?

Find the following limits:

(b) $\lim_{x \to 2} \dfrac{x-2}{4^x - 16}$

(c) $\lim_{t \to 0^+} (1+t)^{6/t}$

(d) $\lim_{x \to 0} \dfrac{\ln(x^2 + x^4)}{\cos(\pi x + \pi)}$

6. (a) Solve the equation $e^{e^x} = 10$ for $x$.

(b) Solve the equation $\log_2(mx) = e$ for $x$.