Practice Problems for Midterm Exam.

1. Given the points $P_1 = (2, 1, 6)$, $P_2 = (4, 7, 9)$, and $P_3 = (11, 7, -12)$.

   a) Find the distance from P_1 to P_2.

   b) Find the vector with length 2 and has opposite direction to the vector $\overrightarrow{P_1P_2}$.

   c) Show that the vector $\overrightarrow{P_1P_2}$ is orthogonal (perpendicular) to the vector $\overrightarrow{P_1P_3}$.

2. a) **True-False** Determine whether the statement is true or false. Explain your answer.

   i) __________ If $\mathbf{v} \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{w}$ and $\mathbf{v} \neq 0$, then $\mathbf{u} = \mathbf{w}$.

   ii) __________ If $\mathbf{u}$ is a unit vector that is parallel to a nonzero vector $\mathbf{v}$, then $\mathbf{u} \cdot \mathbf{v} = \pm \|\mathbf{v}\|$.

   b) Use a determinant to find the cross product

$$\mathbf{i} \times (\mathbf{i} + \mathbf{j} + \mathbf{k})$$

3. a) i) Find parametric equations of the line L passing through the points $P_1(2, 4, -1)$ and $P_2(5, 0, 7)$.

ii) Where does the line intersect the xy-plane?

b) Show that the lines L_1 and L_2 intersect, and find their point of intersection.

$L_1 : x = 1 + 2t, \quad y = 3t, \quad z = 2 - t$

$L_2 : x = -1 + s, \quad y = 4 + s, \quad z = -18 + s.$

4. a) Determine whether the line

$$L : x = 3 - 2t, \quad y = 2 + 6t, \quad z = 5 + 2t$$

is parallel to the plane $x + y - 2z = 9$.

b) Find an equation of the plane that passes through (-1,4,-3) and is perpendicular to the line

$$x = 2 + t, \;\; y = -3 + 3t, \;\; z = -t.$$

5. Find the length of the curve $\mathbf{r}(t) = \langle t^3, t, \frac{\sqrt{6}}{2}t^2 \rangle, \quad 2 \leq t \leq 4.$

Answers

1. a) 7 b) $\dfrac{-2}{7}\langle 2,6,3\rangle$ c) $\overrightarrow{P_1P_2}\cdot\overrightarrow{P_1P_3} = \langle 2,6,3\rangle\cdot\langle 9,6,-18\rangle = 0.$

2. a) F,T b) $-\mathbf{j}+\mathbf{k}$

3. a) *i)* $x = 2+3t,\quad y = 4-4t,\quad z = -1+8t,\quad t\in\mathbb{R}$ (note: the answer can be written in other forms too) ii) $(19/8, 7/2, 0)$
   b) $s = 14,\quad t = 6,\quad$ intersect at $(13, 18, -4)$.

4. a) Method 1: Directional vector of line perpendicular to normal vector of plane, i.e. $\langle -2,6,2\rangle\cdot\langle 1,1,-2\rangle = 0$. Therefore the line parallels to the plane.
   Method 2: plug (x, y, z) of line to equation of the plane to get 5=9. This shows that line never intersects with the plane. Therefore they are parallel to each other.
   b) $x + 3y - z = 14$.

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