Class 8, Practice Problems

Mutivariable Calculus

February 24, 2020

13.2 Limit and Continuity

1. Let $f(x,y) = \dfrac{6x^3y}{2x^4+y^4}$. Find the limit, if it exists, or show that the limit does not exist.

   a) $\lim\limits_{(x,y)\to(1,-2)} f(x,y)$.

   b) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

2. If $f(x,y) = \dfrac{xy\cos y}{3x^2+y^2}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

3. Determine the set of points at which the function is continuous.

   a) $f(x,y) = x^2\sin(xy)$.

   b) $f(x,y) = \ln(x^2+y^2-4)$.

4. Let
$$g(x,y) = \begin{cases} \dfrac{x^2-y^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0). \end{cases}$$
   Is $g(x,y)$ continuous at $(0,0)$?