

Review Calculus

1. Trigonometric functions: $\sin(x)$, $\cos(x)$, $\tan(x)$ where x is in radian.

For examples:

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \quad \tan\left(\frac{\pi}{4}\right) = 1,$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}, \quad \sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{1}{2},$$

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}, \quad \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}.$$

$$\sin(k\pi) = 0 \text{ where } k \in \mathbb{Z},$$

$$\cos(k\pi) = (-1)^k \text{ where } k \in \mathbb{Z}.$$

How to read the values from the unit circle.

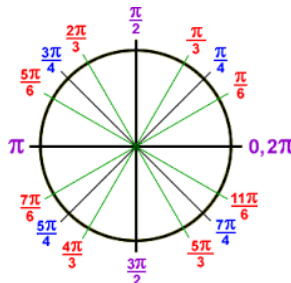


Figure 1: Unit circle with angles measured in radian
(credit: precalculus.blogspot.com/)

For examples,

$$\sin(\pi - x) = \sin(x), \quad \sin(\pi + x) = -\sin(x),$$

$$\cos(\pi - x) = \cos(\pi + x) = -\cos(x).$$

2. Graphing on the xy plane.

Various examples of function $y = f(x)$ and non-function $F(x, y) = c$.

- Straight line: $y = 2x + 1$.
- Exponential: $y = e^x + 1$.
- Parabola: $y = x^2 - 3$.
- Circle: $x^2 + y^2 = 4$.
- Ellipse: $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
- Hyperbola: $x^2 - y^2 = 1$.

3. Solving some simple polynomial equations

For example $x^3 = x$.

4. Calculus: Differentiation

(a) **Trigonometric Differentiation**

$$\begin{aligned}\frac{d}{dx}[\sin(x)] &= \cos(x), \\ \frac{d}{dx}[\cos(x)] &= -\sin(x), \\ \frac{d}{dx}[\tan(x)] &= \sec^2(x) = \frac{1}{\cos^2(x)}.\end{aligned}$$

(b) **Product rule**

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].$$

Example: Calculate $\frac{dy}{dx}$ where $y = x^3 \sin(x)$.

(c) **Chain rule**

Let $y = f(g(x))$ and $u = g(x)$ then $y = f(u)$ and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Examples:

- i. Calculate $\frac{dy}{dx}$ where $y = \cos(\tan x)$.
- ii. Calculate $\frac{dy}{dx}$ where $y = x^3(1+x)^7$.

5. Calculus: Integration

(a) **u -substitution**

$$\int f(g(x))g'(x) dx = \int f(u) du = F(u) + C,$$

where $u = g(x)$ and $du = g'(x)dx$.

Examples:

- i. Evaluate $\int (2x - 30)^{42} dx$.
- ii. Evaluate $\int \frac{x}{(2x^2 + 1)^3} dx$

(b) **integration by parts**

$$\int u dv = uv - \int v du.$$

Example: Evaluate $\int \ln x dx$.

(c) **Trigonometric integration**

Example: Find the area inside the circle radius 1.

Hint: Consider the equation $x^2 + y^2 = 1$ and evaluate

$$\int_{-1}^1 \sqrt{1-x^2} - (-\sqrt{1-x^2}) dx.$$