#Section 2.1.3

```
A := [seq(add(i^2, i = 0..n), n = 0..20)];
GuessPol(A, 0, n);
```

$$A := [0, 1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015, 1240, 1496, 1785, 2109, 2470, 2870]$$

$$\frac{n\,(n+1)\,(2\,n+1)}{6} \tag{1}$$

#Section 2.1.5

```
GenPol(n*(n+1)*(2*n+1)/6, n, x);
```

$$\frac{x^2+x}{(1-x)^4} \tag{2}$$

#Section 2.2.3

```
A := [seq(floor((n/2)^2), n = 0..30)];
GuessC(A, N);
```

$$A := [0, 0, 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49, 56, 64, 72, 81, 90, 100, 110, 121, 132, 144, 156, 169, 182, 196, 210, 225]$$

$$(N+1)\,(N-1)^3 = 0 \tag{3}$$

#Section 2.2.4

```
GenC(N^4-2*N^3+2*N-1, [0, 0, 1, 2], N, x);
```

$$-\frac{x^2}{(x+1)\,(-1+x)^3} \tag{4}$$

#Section 2.2.5

```
A := x^2/(1+x)/(1-x)^3:
B := x/(1-x-x^2):
CAddition(A, B, x);
CCauchy(A, B, x);
CParSum(A, x);
```

$$-\frac{x\left(x^4-x^3+x^2+x-1\right)}{(x+1)\,(-1+x)^3\left(x^2+x-1\right)}$$

$$\frac{x^3}{(x+1)\,(-1+x)^3\left(x^2+x-1\right)}$$

$$\frac{x^2}{(x+1)\,(-1+x)^4} \tag{5}$$

```
CSubSeq(2, N^4-2*N^3+2*N-1, N);
```

$$(N-1)^3\left(N\,d_4-1\right) = 0 \tag{6}$$

```
CTermWise(N^4-2*N^3+2*N-1, N^2-N-1, N);
```

$$\left(N^2+N-1\right)\left(N^2-N-1\right)^3 = 0 \tag{7}$$

#Section 2.2.6

```
CloseC(N^4-2*N^3+2*N-1, [0, 0, 1, 2], 0, N, n);
```

$$\frac{(-1)^n}{8} + \frac{n^2}{4} - \frac{1}{8} \tag{8}$$

#Section 2.3.3

```
A := [seq(add(1/i, i = 1..n), n = 1..35)];
GuessHo(A, 2, 1, 1, n, N);
```

$$A := \left[1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}, \frac{83711}{27720}, \frac{86021}{27720}, \frac{1145993}{360360}, \frac{1171733}{360360}, \frac{1195757}{360360}, \frac{2436559}{720720}, \frac{42142223}{12252240}, \frac{14274301}{4084080}, \frac{275295799}{77597520}, \frac{55835135}{15519504}, \frac{18858053}{5173168}, \frac{19093197}{5173168}, \frac{444316699}{118982864}, \frac{1347822955}{356948592}, \frac{34052522467}{8923714800}, \frac{34395742267}{8923714800}, \frac{312536252003}{80313433200}, \frac{315404588903}{80313433200}, \frac{9227046511387}{2329089562800}, \frac{9304682830147}{2329089562800}, \frac{290774257297357}{72201776446800}, \frac{586061125622639}{144403552893600}, \frac{53676090078349}{13127595717600}, \frac{54062195834749}{13127595717600}, \frac{54437269998109}{13127595717600}\right]$$

$$n + 1 + (-2n - 3)N + (n + 2)N^2 = 0 \tag{9}$$

#Section 2.3.4

```
HoToDiff(n + 1 - N, [1], n, N, x, D);
```

$$\left(x^2 D + x - 1\right) f(x) = -1 \tag{10}$$

```
HoToDiff(4*n + 2 - (n + 2)*N, [1], n, N, x, D);
```

$$(2x - 1 + x(4x - 1)D) f(x) = -1 \tag{11}$$

```
HoToDiff(n + 2 + 2*N - N^2*n, [0, 0], n, N, x, D);
HoToDiff(1 - 2*N + 2*N^3 - N^4, [0, 0, 1, 2], n, N, x, D);
```

$$\left(2x^2 + 2x + 2 + x(-1 + x)(x + 1)D\right) f(x) = 0$$

$$(x + 1)(-1 + x)^3 f(x) = -x^2 \tag{12}$$

```
HoToDiffHom(n + 1 - N, [1], n, N, x, D);
```

$$\left(1 + (3x - 1)D + x^2 D^2\right) f(x) = 0 \tag{13}$$

#Section 2.3.5

```
R1 := 2 + 4*n + (-2 - n)*N:
R2 := n + 1 + (-3 - 2*n)*N + (2 + n)*N^2:
HoAdd(R1, R2, n, N, c);
```

$$-2(n + 1)c_3(3n + 7)(2n + 1)(n + 2)^2 + (3n + 5)(n + 2)\left(9n^3 + 43n^2 + 58n + 20\right)c_3 N - \left(18n^4 + 111n^3 + 241n^2 + 216n + 64\right)c_3 (n + 3)N^2 + c_3 (n + 3)(n + 4)(3n + 4)(n + 1)^2 N^3 = 0 \tag{14}$$

```
R1 := 2 + 4*n + (-2 - n)*N:
R2 := n + 1 + (-3 - 2*n)*N + (2 + n)*N^2:
HoTermWise(R1, R2, n, N, c);
```

$$4(2n + 3)(2n + 1)(n + 1)c_2 - 2(2n + 3)^2 c_2 (n + 2)N + c_2 (n + 2)^2 (n + 3)N^2 = 0 \tag{15}$$

```
DA := lhs(HoToDiffHom(4*n + 2 - (n + 2)*N, [1], n, N, x,
```

```
D)) /f(x);
DB := lhs(HoToDiffHom(n + 1−N, [1], n, N, x, D))
/f(x);
HoCauchy(DA, DB, x, D, c);
```

$$DA := 2 + (10x - 2)\,D + x(4x - 1)\,D^2$$

$$DB := 1 + (3x - 1)\,D + x^2 D^2$$

$$\begin{aligned}&\left(2\left(72x^6 + 660x^5 - 1392x^4 + 900x^3 - 266x^2 + 37x - 2\right)c_4\,(4x - 1) + 2\left(1512x^8\right.\right.\\&\quad + 11076x^7 - 26812x^6 + 22170x^5 - 9442x^4 + 2333x^3 - 342x^2 + 28x - 1\left.\right)c_4\,D\\&\quad + c_4\,x\left(2960x^8 + 19032x^7 - 44104x^6 + 34756x^5 - 13994x^4 + 3230x^3 - 435x^2 + 32x\right.\\&\quad - 1\left.\right)D^2 + 2\left(104x^6 + 636x^5 - 1154x^4 + 654x^3 - 170x^2 + 21x - 1\right)c_4\,x^3\,(4x - 1)\,D^3\\&\quad + c_4\,x^5\,(4x - 1)^2\left(4x^4 + 24x^3 - 31x^2 + 10x - 1\right)D^4\left.\right)f(x) = 0\end{aligned} \tag{16}$$

```
#Section 3.4
C2ToDiff(N−(c1*a^(n + 2) + c2*b^(n + 2)), {1, a, b},
[a0], n, N, x, D);
```

$$f(x) - c1\,a^2\,x f(a\,x) - c2\,b^2\,x f(b\,x) = a0$$

$$[[1, a, b], [1, -c1\,a^2\,x, -c2\,b^2\,x], a0] \tag{17}$$

```
C2ToDiff(N−(n + 1)*2^n, {1, 2}, [a0], n, N, x, D);
```

$$f(x) + \left(-x^2\,D - x\right)f(2\,x) = a0$$

$$[[1, 2], [1, -x^2\,D - x], a0] \tag{18}$$

```
C2ToDiff(N^2−N−2^n, {1, 2}, [a0, a1], n, N, x, D);
```

$$(1 - x)\,f(x) - x^2 f(2\,x) = -a0\,x + a1\,x + a0$$

$$[[1, 2], [1 - x, -x^2], -a0\,x + a1\,x + a0] \tag{19}$$

```
C2ToDiffHom(N−(c1*a^(n + 2) + c2*b^(n + 2)), {1, a, b},
[a0], n, N, x, D);
```

$$-D\,a0\,f(x) + \left(c1\,a^2\,x\,a0\,D + c1\,a^2\,a0\right)f(a\,x) + \left(c2\,b^2\,x\,a0\,D + c2\,b^2\,a0\right)f(b\,x) = 0$$

$$[[1, a, b], [-D\,a0, c1\,a^2\,(D\,x + 1)\,a0, c2\,b^2\,(D\,x + 1)\,a0], 0] \tag{20}$$

```
#Section 3.5
HoAdd(N−F(n + 2), N^2−N−2^n, n, N, c);
HoTermWise(N−F(n + 2), N^2−N−2^n, n, N, c);
```

$$\begin{aligned}&-2^n\,F(n + 2)\left(-F(n + 3)\,F(n + 4) + 2\,2^n + F(n + 3)\right)c_3 + \left(-2\,F(n + 2)\,F(n + 3)\,2^n\right.\\&\quad + F(n + 2)\,F(n + 3)\,F(n + 4) + 2\left(2^n\right)^2 - F(n + 2)\,F(n + 3)\left.\right)c_3\,N + \left(-F(n\right.\\&\quad + 2)\,F(n + 3)\,F(n + 4) + 2\,F(n + 2)\,2^n + 2^n + F(n + 2)\left.\right)c_3\,N^2 - c_3\left(-F(n\right.\\&\quad + 2)\,F(n + 3) + 2^n + F(n + 2)\left.\right)N^3 = 0\end{aligned}$$

$$-c_2\,F(n + 2)\,F(n + 3)\,2^n - F(n + 3)\,c_2\,N + c_2\,N^2 = 0 \tag{21}$$